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Generation of superpositions of coherent states on a circle

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Abstract. We propose a simple method to obtain a superposition of coherent states on a circle, including Schrödinger cat states as a special case, *via* conditional measurement of the state of three level atoms interacting with a one mode cavity field. In the low amplitude limit, very good approximation of Fock states can also be generated in this way.

PACS. 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phase measurements

1 Introduction

The possibility of accurate manipulation of a cavity field and the high level of control of its interaction with matter have lead to the production of photon states with many non-classical features, such as photon antibunching and various forms of squeezing [1]. In particular, many theoretical schemes have been proposed to generate superpositions of macroscopically distinguishable (coherent) states (the so-called "Schrödinger Cats") [2,4], and photon number state [5]. Even and odd coherent states, for example, can be produced during the time evolution of a Jaynes-Cummings Model with large amplitude initial fields at half the revival time [3], or through the dispersive atom-field coupling realizing a Quantum Nondemolition Measurement scheme for the photon number distribution [6]. In the latter, a Fock state can also be obtained after many atoms are let to interact with the field one at a time, leading to gradual diffusion of the field phase and decimation of the photon number distribution.

The general idea of using atoms to manipulate the electromagnetic field state of a micromaser cavity has been employed in several schemes, often in connection with state selective measurement. The measurement has to be performed on each atom exiting the cavity in order to "reduce" the interaction generated atom-field entangled state; the projection of the field onto the desired state can be achieved if the proper sequence of measurement results is obtained. Based on the same idea, we propose the use of a three level atom in the V configuration to generate a class of linear superpositions of coherent states on a circle, *i.e.* coherent states with the same modulus but different phases. It has been shown that certain quantum states and particularly pure number ones, can be arbitrarily well-approximated through such discrete superpositions, provided the circle has a small enough radius [7].

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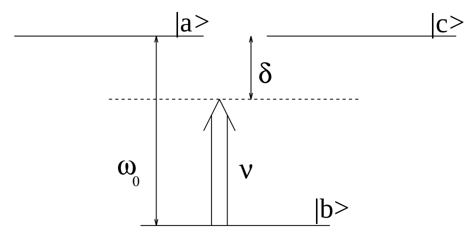

Fig. 1. Level scheme of a V-type atom with two degenerate upper states, interacting with a cavity field of frequency ν detuned from resonance of an amount $\delta = \omega_0 - \nu$.

As we will show in the following, the scheme we propose is well-suitable for this task.

2 The model

Let us consider the atomic configuration schematically represented in Figure 1. We suppose the V-type three level atom to have degenerate upper levels, $|a\rangle$ and $|c\rangle$, and to interact with a cavity mode that couples both of them to the ground state. Defining $\delta = \omega_0 - \nu$ as the detuning between the atomic transition frequency and the field frequency and g_1 and g_2 as the vacuum Rabi frequencies corresponding to the two transitions, we can write the Hamiltonian for this system as

$$H = H_0 + H_I$$

with

$$H_0 = \omega_0|a\rangle\langle a| + \omega_0|c\rangle\langle c| + \nu a^\dagger a \tag{1}$$

$$H_I = (g_1|a\rangle\langle b|a + h.c.) + (g_2|c\rangle\langle b|a + h.c.)\,. \tag{2}$$

If the interaction time is much lower than both the atomic excited states and cavity lifetimes, all the incoherent processes can be neglected.

$$|\psi,t\rangle = \sum_n w_n \left\{ \frac{A|g_1|^2 + Cg_1g_2^*}{\lambda_{+,n}^2 + G^2(n+1)}(n+1)e^{-i\lambda_{+,n}t} + \frac{A|g_2|^2 - Cg_1g_2^*}{G^2} + \frac{A|g_1|^2 + Cg_1g_2^*}{\lambda_{-,n}^2 + G^2(n+1)}(n+1)e^{-i\lambda_{-,n}t} \right\} e^{-i(\omega_0+n\nu)t}|a,n\rangle$$
$$+ \sum_n w_n \left\{ \frac{\lambda_{+,n}(Ag_1^* + Cg_2^*)\sqrt{n+1}}{\lambda_{+,n}^2 + G^2(n+1)} e^{-i\lambda_{+,n}t} + \frac{\lambda_{-,n}(Ag_1^* + Cg_2^*)\sqrt{n+1}}{\lambda_{-,n}^2 + G^2(n+1)} e^{-i\lambda_{-,n}t} \right\} e^{-i(n+1)\nu t}|b,n+1\rangle$$
$$+ \sum_n w_n \left\{ \frac{(Ag_1^*g_2 + C|g_2|^2)(n+1)}{\lambda_{+,n}^2 + G^2(n+1)} e^{-i\lambda_{+,n}t} - \frac{Ag_1^*g_2 + C|g_1|^2}{G^2} + \frac{(Ag_1^*g_2 + C|g_2|^2)(n+1)}{\lambda_{-,n}^2 + G^2(n+1)} e^{-i\lambda_{-,n}t} \right\} e^{-i(\omega_0+n\nu)t}|c,n\rangle \quad (4)$$

$$|\psi,t\rangle \simeq \sum_n w_n \left\{ \frac{A|g_1|^2 + Cg_1g_2^*}{G^2} e^{-i\frac{G^2(n+1)}{\delta}t} + \frac{A|g_2|^2 - Cg_1g_2^*}{G^2} \right\} e^{-i(\omega_0+n\nu)t}|a,n\rangle$$
$$+ \sum_n w_n \sqrt{n+1}\frac{Ag_1^* + Cg_2^*}{\delta} \left\{ e^{-i\frac{G^2(n+1)}{\delta}t} - e^{-i\delta t} \right\} e^{-i(n+1)\nu t}|b,n+1\rangle$$
$$+ \sum_n w_n \left\{ \frac{Ag_1^*g_2 + C|g_2|^2}{G^2} e^{-i\frac{G^2(n+1)}{\delta}t} - \frac{Ag_1^*g_2 - C|g_2|^2}{G^2} \right\} e^{-i(\omega_0+n\nu)t}|c,n\rangle \quad (6)$$

With this assumption and choosing the initial condition as

$$|\psi,0\rangle = \left(\sum_n w_n|n\rangle\right) \otimes (A|a\rangle + C|c\rangle) \quad (3)$$

we can readily obtain the state vector at any subsequent time t:

see equation (4) above

where we have defined $G^2 = |g_1|^2 + |g_2|^2$, and

$$\lambda_{\pm,n} = -\frac{\delta}{2} \pm \left[\frac{\delta^2}{4} + G^2(n+1)\right]^{\frac{1}{2}} \quad (5)$$

If the condition $\delta \gg G(\bar{n}+1)^{1/2}$ is fulfilled, $\bar{n}$ being the mean photon number of the field, the preceding expression can be approximated as

see equation (6) above.

As expected, the probability to find the atom in its ground state is very small due to the large detuning.

Supposing the field to be initially in a coherent state, $\sum_n w_n|n\rangle = |\alpha\rangle$, the atom to be detected just outside the cavity and found to be in the state $|a\rangle$ or $|c\rangle$, the field state is reduced to either

$$|\psi_{field}^{(1)}\rangle_{(a)} = \frac{e^{-i\omega_0 t}}{\sqrt{P_a}} \left[\frac{A|g_1|^2 + Cg_1g_2^*}{G^2} e^{-i\theta}|\alpha e^{-i(\nu t+\theta)}\rangle + \frac{A|g_2|^2 - Cg_1g_2^*}{G^2}|\alpha e^{-i\nu t}\rangle \right] \quad (7)$$

or

$$|\psi_{field}^{(1)}\rangle_{(c)} = \frac{e^{-i\omega_0 t}}{\sqrt{P_c}} \left[\frac{Ag_1^*g_2 + C|g_2|^2}{G^2} e^{-i\theta}|\alpha e^{-i(\nu t+\theta)}\rangle - \frac{Ag_1^*g_2 - C|g_1|^2}{G^2}|\alpha e^{-i\nu t}\rangle \right] \quad (8)$$

respectively.

Here, P_a and P_c are the probability of occupation of the two upper levels when the atom leaves the interaction region, while the phase θ is given by

$$\theta = \frac{G^2}{\delta}t.$$

We see that in both cases the field state is a superposition of two coherent states with the same modulus, $|\alpha|$, but with a phase difference given by θ. It can be noted, further, that $|\psi_{field}^{(1)}\rangle_{(a)}$ and $|\psi_{field}^{(1)}\rangle_{(c)}$ are distinct linear superpositions of the same two states; we will take advantage of this fact when describing the production of an even coherent state in the following section.

For the sake of simplicity, from now on we suppose that $g_1 = g_2$. This assumption greatly simplifies the algebra, but all the superpositions we indicate in the following can still be obtained when the two coupling constants have different values.

3 States obtained by repeated atom-field interaction

First, we note that, apart from the phase rotation of νt due to the free field Hamiltonian evolution, the first term of the superposition states obtained after the passage of the first atom exactly reproduces the initial cavity state. This suggests that, after the passage of a second atom with the same velocity, we could find the field in a linear combination of three coherent states, the third one having a phase difference of 2θ from the initial one. This is indeed the case.

More generally, if we suppose that m atom in the same internal state are sent through the cavity with a well-selected velocity so as to have the same interaction time and, further, that they are all found to be in the state $|a\rangle$ after the interaction, then, the field state will be

$$|\psi^{(m)}_{field}\rangle_{m\times a} = N_m \sum_{k=0}^{m} \binom{m}{k} (A-C)^{m-k} \times (A+C)^k |\alpha e^{-i(\nu T + k\theta)}\rangle \qquad (9)$$

where N_m is a normalization constant, while T is the total elapsed time from the entrance of the first atom to the exit of the last one. If the atom injection rate is chosen so that the cavity is never empty, T is given by mt.

As another example, we can look at the state obtained when two atoms with the same initial state are detected one in level $|a\rangle$ and the other in level $|c\rangle$ after the transit time. In this case

$$|\psi^{(2)}_{field}\rangle_{a,c} = N_2 \left\{ -(A-C)^2 |\alpha e^{-2i\nu t}\rangle + (A+C)^2 \exp(-2i\theta) |\alpha e^{-2i(\nu t+\theta)}\rangle \right\}. \qquad (10)$$

It is easy to see that this gives an even coherent state provided that $\theta = \pi/2$ and, for example, $A = 1, C = 0$.

If four atoms are sent, and two of them are detected in $|a\rangle$, the other two in $|c\rangle$, we have

$$|\psi^{(4)}_{field}\rangle_{2a,2c} = N_4 \Big\{ (A-C)^4 |\alpha e^{-4i\nu t}\rangle - 2(A^2-C^2)^2 e^{-2i\theta} \times |\alpha e^{-i(4\nu t+2\theta)}\rangle + (A+C)^4 e^{-4i\theta} |\alpha e^{-4i(\nu t+\theta)}\rangle \Big\}. \qquad (11)$$

It can be noted that both these results, equations (10, 11), are independent from the detection order; that is, the order in which detection outcomes are found is irrelevant, one needs only that half of the atoms has been found in $|a\rangle$, the other half in $|c\rangle$.

3.1 Production of an approximate number state

In all preceding examples, atoms were supposed to be injected always with the same initial state, given in equation (3). Nothing forbids employing atoms with differently prepared internal states, and this turns out to be the key for the production of an approximate number state. As clearly described in reference [7], the number state $|n\rangle$ can be very well-approximated by a discrete superposition of $(n+1)$ equidistant coherent states on a circle, provided that the chosen circle has a small radius. In particular, the superposition

$$\sum_{k=0}^{n} \exp\left(\frac{2\pi i}{n+1} k\right) |\alpha e^{\frac{2\pi i}{n+1} k}\rangle \qquad (12)$$

where α can be any complex number, rapidly converges to the Fock state $|n\rangle$ (apart from normalization) when $|\alpha|$ becomes small.

We note that the structure of this superposition requires the presence of the factor $e^{i\theta}$ in front of the θ dephased coherent state. This precisely happens for the superpositions generated in scheme proposed in the previous section, which is, thus, clearly suitable for this task.

It is easy to see, however, that a superposition like (12) cannot be produced if the atoms all have the same initial state, whatever the detection sequence may be. Different preparations of the internal state are therefore required for the atoms. The coefficients $\{A_i, C_i;\ i = 1, \ldots, n\}$ have to be determined accordingly to the chosen value of n. As an example, if the state with exactly three photons has to be prepared, we need three atoms to interact with the cavity field, initially prepared in a coherent state of small amplitude. In general, the accuracy of the approximation with the decreasing modulus of $|\alpha|$ is given by

$$\frac{n!}{(2n+1)!} |\alpha|^{2(n+1)} + o\left(|\alpha|^{4(n+1)}\right)$$

and $|\alpha| \le 0.5$ suffices in this case, see reference [7].

The three atoms should be prepared in the following initial states:

$$|\psi_{atom}\rangle_1 = \frac{1}{\sqrt{2}} (|a\rangle + i|c\rangle)$$
$$|\psi_{atom}\rangle_2 = \frac{1}{\sqrt{2}} (|a\rangle - i|c\rangle)$$
$$|\psi_{atom}\rangle_3 = |a\rangle.$$

The injection order is irrelevant, but all the atoms need to be detected in state $|a\rangle$ after the interaction. Another, obviously necessary condition is that the interaction time has to be so chosen such that $\theta = \pi/2$.

The final field state is given by

$$|\psi_{field}\rangle = N' \Big\{ |\tilde{\alpha}\rangle + i|i\tilde{\alpha}\rangle - |-\tilde{\alpha}\rangle - i|-i\tilde{\alpha}\rangle \Big\}$$

where $\tilde{\alpha} = \alpha e^{-3i\nu t}$.

We note that this state is a four photon coherent state of the kind studied in reference [8]. They can be produced for arbitrary value of $|\alpha|$, but we illustrate in detail the case of small amplitude because of its connection with the Fock state. Indeed, the generation of the approximate number state can be directly followed in phase space. Figures 2 and 3 report the evolution of the cavity field Q-function after the injections of the three atoms in the previously indicated states for an initially prepared coherent state with $\alpha = 0.5$. The initial Gaussian shaped function describing the coherent state first shifts toward the center, then produces a dip there, becoming more and more symmetric, until it is practically indistinguishable from that of a number state after the injection of the third atom.

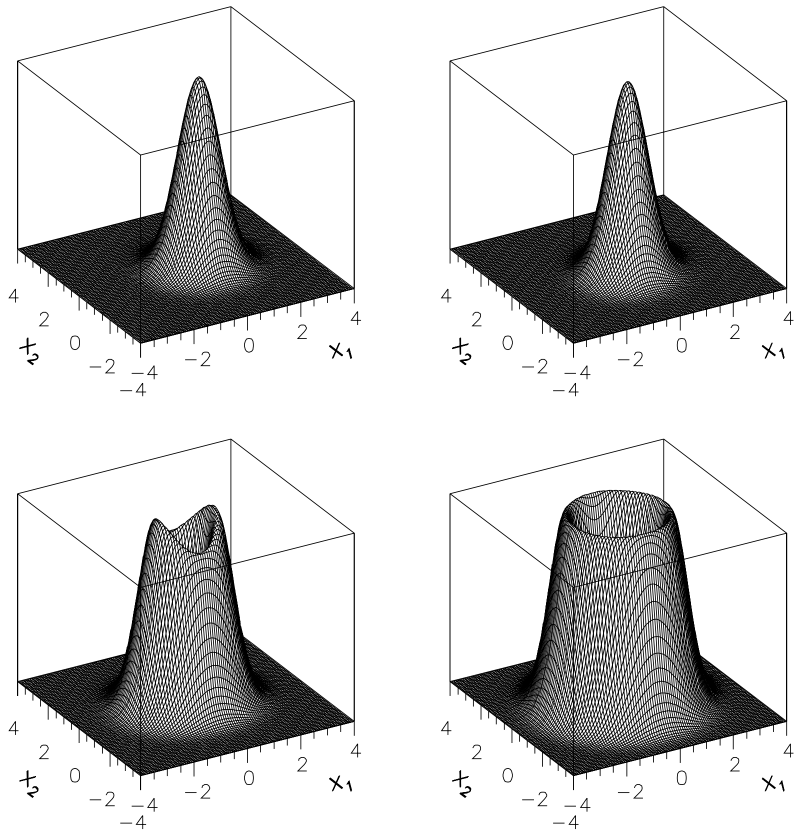

Fig. 2. The Q-function of the field after the injection of the three atoms properly prepared to produce the Fock state $|3\rangle$.

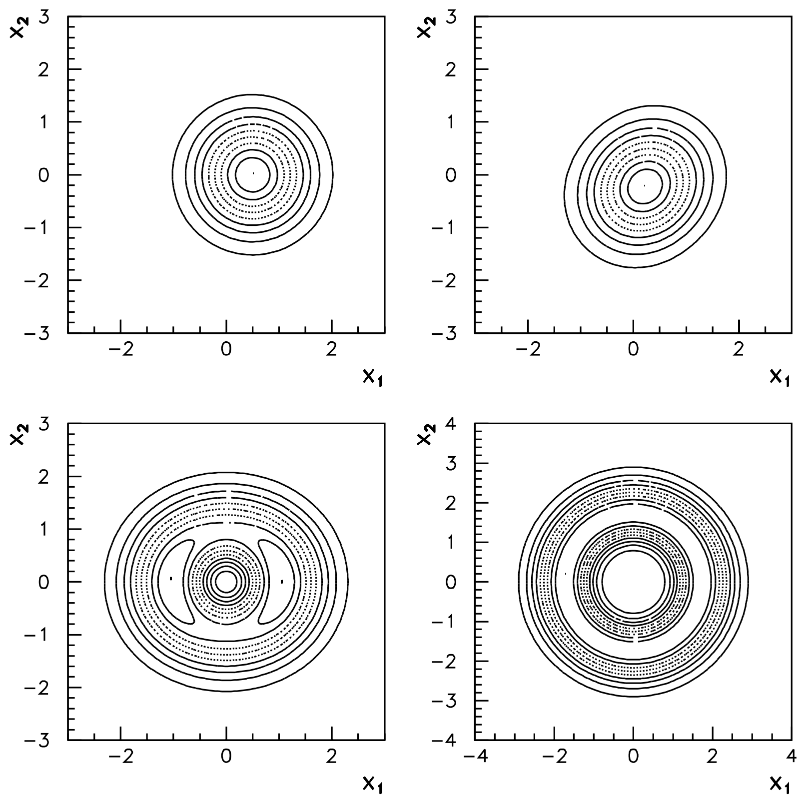

Fig. 3. Contour plot for the Q-functions of Figure 2.

4 Conclusions

We have presented an atom-field interaction based scheme for the production of a large class of superpositions of coherent states on a circle whose radius is determined by the initial field state. The proposed scheme, which requires a dispersive, non-resonant interaction between a three level atom in the V configuration and a cavity field initially prepared in a coherent state, offers the possibility of detecting the atoms in both their excited states, still giving combinations of states with a given phase shift. This can be very useful for the preparation of selected superpositions. Moreover, using atoms prepared in different initial states, a very large class of field states can be generated, including approximate realization of number states, obtained through a discrete number of coherent states placed on a circle of small radius.

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